8. Hacemos el cambio de variable $x+2=t \rightarrow t=x-2$, $dt=dx$:

$$\int \frac{x}{(x+2)^2}dx = \int \frac{t-2}{t}dt = \int \frac{1}{t}dt + \int \frac{2}{t^2}dt = \ln(t) + \frac{2}{t} = \ln(x+2) + \frac{2}{x+2} \tag{1}$$

11. Hacemos el cambio de variable $x=t^2 \rightarrow dx = 2tdt$, cambiando los límites a: $x=0 \Rightarrow t=0$ y $x=1 \Rightarrow t=+1$:

$$\int_0^1 \left(1+\sqrt{x}\right)^8 dx = \int_0^1 (1+t)^8\, 2tdt = 2\int_0^1 \left(t+8t^2+28t^3+56t^4+70t^5+56t^6+28t^7+8t^8+t^9\right) dt \tag{2}$$

Resolviendo la integral que ha quedado:

$$\int_0^1 \left(1+\sqrt{x}\right)^8 dx = 2\left[\frac{1}{2}t^2+\frac{8}{3}t^3+7t^4+\frac{56}{5}t^5+\frac{35}{3}t^6+8t^7+\frac{7}{2}t^8+\frac{8}{9}t^9+\frac{1}{10}t^9\right]_0^1 = \frac{4097}{45} \tag{3}$$

14. Hacemos el cambio de variable $x = \operatorname{sen}\theta \Rightarrow dx = \cos\theta d\theta$:

$$\int x \operatorname{sen}^{-1} x = \int x \operatorname{arc\,sen} x = \int \operatorname{sen}\theta\cos\theta\theta d\theta = \frac{1}{2}\int \operatorname{sen}(2\theta)\theta d\theta \tag{4}$$

Y ahora integramos por partes ($u=\theta$, $dv = \operatorname{sen}\theta d\theta$):

$$\begin{aligned} &\frac{1}{2}\left(\frac{\theta}{2}\cos(2\theta) - \frac{1}{2}\int \cos(2\theta)\, d\theta\right) = \\ &= \frac{1}{4}\left(\theta\cos(2\theta) - \frac{1}{2}\operatorname{sen}(2\theta)\right) = \end{aligned} \tag{5}$$

Deshaciendo le cambio de variable:

$$= \frac{1}{4}\left[\operatorname{arc\,sen} x\left(1-2x^2\right) - x\sqrt{1-x^2}\right] \tag{6}$$

17. Integramos por partes ($u=x^2$,$dv = \cosh(x)dx$)

$$\int x^2\cosh x = x^2\operatorname{senh} x - 2\int x\operatorname{senh} x dx = \left(x^2+2\right)\operatorname{senh} x - 2x\cosh x \tag{7}$$

Done hemos vuelto a integrar por partes ($u=x$,$dv=\operatorname{senh}(x)dx$).

20 Hacemos el cambio de variable $x=t^2$:

$$\int \cos\left(\sqrt{x}\right)dx = 2\int t\cos(t)\,dt = 2\left(t\operatorname{sen} t + \cos t\right) = 2\left(\sqrt{x}\operatorname{sen}\sqrt{x} + \cos\sqrt{x}\right) \tag{8}$$

26. Integramos por partes ($dv = x^2$, $u = \ln(1+x)$):

$$\begin{aligned}\int x^2 \ln(1+x)\,dx &= \frac{x^3}{3}\ln(1+x) - \frac{1}{3}\int\left(\frac{x^3}{1+x}dx\right) = \\ &= \frac{x^3}{3}\ln(1+x) - \frac{1}{3}\int\left(x^2 - x + 1 - \frac{1}{1+x}dx\right) = \\ &= \frac{x^3}{3}\ln(1+x) - \frac{1}{3}\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 + x - \ln(1+x)\right) = \\ &= \frac{1}{18}\left[6\left(x^3+1\right)\ln(1+x) + x\left(2x^2 - 3x^3 - 6x\right)\right]\end{aligned} \tag{9}$$

29. Completamos cuadrados en el radicando:

$$\begin{aligned}\int \frac{1}{\sqrt{9x^2+12x-5}}\,dx &= \int \frac{1}{\sqrt{(3x+2)^2-9}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{3x+2}{3}\right)^2-1}}\,dx = \\ &= \frac{1}{3}\int \frac{1}{\sqrt{\left(x+\frac{2}{3}\right)^2-1}}\,dx = \frac{1}{3}\,\text{arccosh}\left(x+\frac{2}{3}\right)\end{aligned} \tag{10}$$

35. Utilizamos las relaciones de ángulo doble:

$$\sin^2(2x) = \frac{1-\cos(4x)}{2}, \quad \cos^2(x) = \frac{1+\cos(2x)}{2}, \quad \text{sen}(2x) = 2\,\text{sen}\,x\cos(x) \tag{11}$$

$$\begin{aligned}\int \sin^2 x\cos^4 x dx &= \frac{1}{4}\int\left(4\,\text{sen}^2 x\cos^4 x\right)dx = \frac{1}{4}\int \text{sen}^2(2x)\cos^2 x dx = \\ &= \frac{1}{4}\int \text{sen}^2(2x)\left(\frac{1+\cos(2x)}{2}\right)dx = \frac{1}{8}\int\left(\text{sen}^2(2x) + \text{sen}^2(2x)\cos(2x)\right) = \\ &= \frac{1}{8}\int \text{sen}^2(2x)dx + \frac{1}{8}\int \text{sen}^2(2x)\cos(2x)\,dx = \\ &= \frac{1}{16}\int(1-\cos(4x))\,dx + \frac{1}{48}\text{sen}^3(2x) = \\ &= \frac{1}{48}\text{sen}^3(2x) + \frac{1}{16}\left(x + \frac{1}{4}\text{sen}(4x)\right)\end{aligned} \tag{12}$$

38. Utilizamos las relaciones del ángulo medio:

$$1+\cos x = 2\cos^2\left(\frac{x}{2}\right), \quad \text{sen}\,x = 2\,\text{sen}\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) \tag{13}$$

$$\begin{aligned}\int \frac{1+\cos x}{\text{sen}\,x}dx &= \int \frac{\cos^2\left(\frac{x}{2}\right)}{\text{sen}\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}dx = \\ &= \int \frac{\cos\left(\frac{x}{2}\right)}{\text{sen}\left(\frac{x}{2}\right)}dx = \ln\left(\text{sen}\left(\frac{x}{2}\right)\right)\end{aligned} \tag{14}$$

41. Integramos por partes ($u = x^5$, $dv = \cosh x dx$):

$$\begin{aligned}\int_{-1}^{1} \cosh x dx &= \left(x^5 \sinh x\right)_{-1}^{1} - 5\int_{-1}^{1} x^4 \operatorname{senh} x dx = -5\int_{-1}^{1} x^4 \operatorname{senh} x dx = \\ &= -5\left(x^4 \cosh x\right)_{-1}^{1} + \int_{-1}^{1} 20x^3 \cosh x dx = \int 20x^3 \cosh x dx = \\ &= 20\left(x^3 \operatorname{senh} x\right)_{-1}^{1} - \int_{-1}^{1} 60 \operatorname{senh} x x^2 dx = -\int_{-1}^{1} 60 \operatorname{senh} x x^2 dx = \\ &= -\left(x^2 \cosh x\right)_{-1}^{1} + \int 120x \cosh x dx = \int 120x \cosh x dx = \\ &= \left(x \operatorname{senh} x\right)_{-1}^{1} - \left(120 \cosh x\right)_{-1}^{1} = 0\end{aligned} \tag{15}$$

44.

$$\begin{aligned}\int_{0}^{\pi/4} \cos^5 \theta d\theta &= \int_{0}^{\pi/4} \cos^4 \theta \cos \theta d\theta = \int_{0}^{\pi/4} \left(1 + \operatorname{sen}^4 \theta - 2\operatorname{sen}^2 \theta\right) \cos \theta d\theta = \\ &= \int_{0}^{\pi/4} \cos \theta d\theta + \int_{0}^{\pi/4} \operatorname{sen}^4 \theta \cos \theta d\theta - 2\int \operatorname{sen}^2 \theta \cos \theta d\theta = \\ &= \left(\operatorname{sen} \theta\right)_{0}^{\pi/4} + \left(\frac{1}{5}\operatorname{sen}^5 \theta\right)_{0}^{\pi/4} - 2\left(\frac{1}{3}\operatorname{sen}^3 \theta\right)_{0}^{\pi/4} = \\ &= \frac{1}{\sqrt{2}} + \frac{1}{20\sqrt{2}} - \frac{1}{3\sqrt{2}} = \frac{43}{60\sqrt{2}}\end{aligned} \tag{16}$$

47.

$$\begin{aligned}\int \frac{x}{\left(x^2+1\right)\left(x^2+4\right)} dx &= \frac{1}{3}\int \left(\frac{x}{x^2+1} - \frac{x}{x^2+4}\right) dx = \frac{1}{3}\int \left(\frac{1}{2}\ln\left(x^2+1\right) - \frac{1}{2}\ln\left(x^2+4\right)\right) = \\ &= \frac{1}{6}\ln\left(\frac{x^2+1}{x^2+4}\right)\end{aligned} \tag{17}$$

50. Hacemos el cambio de variable $x = t^3$:

$$\begin{aligned}\int e^{\sqrt[3]{x}} dx &= 3\int e^t t^2 dt = 3\left(t^2 e^t - 2\int t e^t dt\right) = 3\left(t^2 e^t - 2\left(te^t - e^t\right)\right) = \\ &= 3t^2 e^t - 6te^t + 6e^t =\end{aligned} \tag{18}$$

Deshaciendo el cambio de variable:

$$= e^{\sqrt[3]{x}}\left(3x^{2/3} - 6x^{1/3} + 6\right) \tag{19}$$

53. Integramos por partes ($u = x^2 + 4x - 3$, $dv = \operatorname{sen}(2x)\, dx$):

$$\int \left(x^2+4x-3\right)\operatorname{sen}(2x)\,dx = -\frac{1}{2}\left(x^2+4x-3\right)\cos(2x)+\frac{1}{2}\int (2x+4)\cos(2x)\,dx = \tag{20}$$

Volvemos a aplicar integración por partes ($u = 2x+4$, $dv = \cos(2x)\,dx$)

$$\begin{aligned} &= -\frac{1}{2}\left(x^2+4x-3\right)\cos(2x)+\frac{1}{4}(2x+4)\operatorname{sen}(2x)+\frac{1}{4}\cos(2x) = \\ &= \frac{1}{4}\left[\left(7-8x-2x^2\right)\cos(2x)+(2x+4)\operatorname{sen}(2x)\right] \end{aligned} \tag{21}$$

59. Esta es inmediata:

$$\int \frac{e^{\arctan x}}{1+x^2}dx = e^{\arctan x} \tag{22}$$

62. Hacemos el cambio de variable $t = x^6$:

$$\begin{aligned} \int \frac{\sqrt{t}}{1+\sqrt[3]{t}}dt = 6\int \frac{x^8}{1+x^2}dx = 6\int\left(x^6-x^4+x^2-1+\frac{1}{1+x^2}\right)dx = \\ = 6\left(\frac{x^7}{7}-\frac{x^5}{5}+\frac{x^3}{3}-x+\arctan x\right) = \end{aligned} \tag{23}$$

Deshaciendo el cambio de variable:

$$= 6\left(\frac{t^{7/6}}{7}-\frac{t^{5/6}}{5}+\frac{t^{3/6}}{3}-t^{1/6}+\arctan t^{1/6}\right) \tag{24}$$

65.

$$\begin{aligned} \int \sqrt{\frac{1+x}{1-x}} = \int \frac{1+x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{x}{\sqrt{1-x^2}}\,dx = \\ = \operatorname{arc\,sen} x - \sqrt{1-x^2} \end{aligned} \tag{25}$$

68. Completamos cuadrados:

$$\begin{aligned} \int \sqrt{1+x-x^2}dx = \int \sqrt{1+\frac{1}{4}-\left(\frac{1}{4}-x+x^2\right)}dx = \int \sqrt{\frac{5}{4}-\left(x-\frac{1}{2}\right)^2}dx = \\ = \frac{\sqrt{5}}{2}\int \sqrt{1-\left[\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)\right]^2}dx = \end{aligned} \tag{26}$$

Hacemos el cambio de variable $\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)=\operatorname{sen}\theta$

$$\begin{aligned} &= \frac{5}{4}\int \cos^2\theta d\theta = \frac{5}{4}\int\left(\frac{1+\cos\theta}{2}\right)d\theta = \\ &= \frac{5}{8}\left(\theta+\frac{1}{4}\operatorname{sen}(2\theta)\right) = \end{aligned} \tag{27}$$

Deshaciendo el cambio de variable:

$$\begin{aligned} &= \frac{5}{8}\left[\operatorname{arc\,sen}\left[\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)\right]+\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)\sqrt{1-\left(\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)\right)^2}\,\right] = \\ &= \frac{5}{8}\operatorname{arc\,sen}\left[\frac{2}{\sqrt{5}}\left(x-\frac{1}{2}\right)\right]+\frac{1}{2}\left(x-\frac{1}{2}\right)\sqrt{1+x-x^2} \end{aligned} \tag{28}$$

71. Hacemos primero integración por partes ($u=x$, $dv=\frac{\operatorname{sen}x}{\cos^2 x}dx$):

$$\int x\sec x\tan x dx = \int x\frac{\operatorname{sen}x}{\cos^2 x}dx = \frac{x}{\cos x}-\int\frac{1}{\cos x}dx = \tag{29}$$

Ahora hacemos el cambio $x=\arctan\theta$:

$$= \frac{x}{\cos x}-\int\frac{\sqrt{1+\theta^2}}{1+\theta^2}d\theta = \frac{x}{\cos x}-\operatorname{arcsenh}(\tan x) \tag{30}$$

74. Hacemos el cambio de variable $e^x=t$:

$$\begin{aligned} \int\frac{1}{1+2e^x-e^{-x}}dx &= \int\frac{e^x}{e^x+2e^{2x}-1}dx = \int\frac{1}{t+2t^2-1}dt = \\ &= \frac{1}{2}\int\frac{1}{\frac{t}{2}+t^2-\frac{1}{2}}dt = \frac{1}{2}\int\frac{1}{\left(t+\frac{1}{4}\right)^2-\frac{9}{16}}dt = \\ &= -\frac{8}{9}\int\frac{1}{1-\left(\frac{4t+1}{3}\right)^2}dt = -\frac{2}{3}\operatorname{arctanh}\left(\frac{4t+1}{3}\right) = \end{aligned} \tag{31}$$

Deshaciendo el cambio de variable:

$$= -\frac{2}{3}\operatorname{arctanh}\left(\frac{4e^x+1}{3}\right) \tag{32}$$

77. Hacemos el cambio de variable $e^{-x}=t$:

$$\begin{aligned} \int\frac{1}{e^{3x}-e^x}dx &= \int\frac{e^{-x}}{e^{2x}-1}dx = -\int\frac{1}{\frac{1}{t^2}-1}dt = -\int\frac{t^2}{1-t^2}dt = \\ &= -\int\left(-1+\frac{1}{1-t^2}\right)dt = -\left(-t+\operatorname{arctanh}(t)\right) = \end{aligned} \tag{33}$$

Deshaciendo el cambio de variable:

$$= e^{-x}-\operatorname{arctanh}\left(e^{-x}\right) \tag{34}$$

80. Completamos cuadrados:

$$\int \frac{\operatorname{sen}(2x)}{\sqrt{9-\cos^4 x}}\,dx = \frac{1}{3}\int \frac{2\operatorname{sen} x\cos x}{\sqrt{1-\frac{1}{9}\cos^4 x}}\,dx = \\ = -\int \frac{-2/3\operatorname{sen} x\cos x}{\sqrt{1-\left(\frac{1}{3}\cos^2 x\right)^2}}\,dx = -\operatorname{arcsenh}\left(\frac{1}{3}\cos^2 x\right) \tag{35}$$